Causal Inference
7 - Synthetic Controls

Benjamin Elsner
benjamin.elsner@ucd.ie

Synthetic Control

Mixtape Ch. 11

Synthetic Controls

The **synthetic control (SC) method** has become **increasingly popular in economics** and other disciplines

Goal: estimate the **causal effect of an event that occurs at an aggregate level** (country, city, state, etc)

- the effect of a change in monetary or fiscal policy on GDP, unemployment, etc
- the effect of conflict on various outcomes
- the effect of law change in one state

Challenges:

- difficult to find a suitable counterfactual
- only one unit is treated $\Rightarrow$ challenging inference

Synthetic Controls

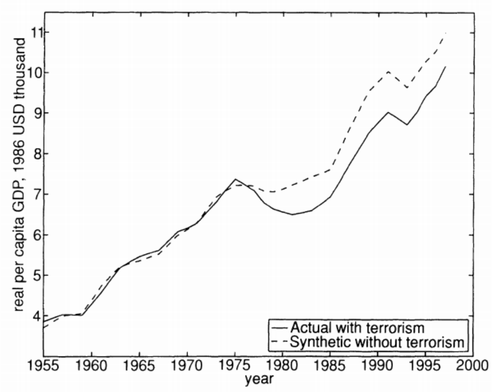
SC is a **difference-in-differences estimator** that is suitable for answering such questions

Main idea: **data-driven counterfactual**

- the **counterfactual is a weighted average** of all potential control units
- the weights are determined by a matching algorithm
- ...chosen to closely **match the trend before the event**

Synthetic Control Example

Classic example for **synthetic controls**: impact of **terrorism in the Basque country on GDP** (Abadie & Gardeazabal, 2003)



Control group: **weighted average of other Spanish citites**

Synthetic Controls: Set-up

We **observe** $J + 1$ **units** in periods $t = 1, \ldots, T$

One unit is exposed to an intervention in $t = \tau$; hence it is treated in all periods after τ

The remaining J units are an **untreated reservoir of potential controls** (**"donor pool"**)

Potential outcomes

- Y_{1t}^{0} outcome of unit i at time t in absence of a treatment
- Y_{1t}^{1} otucomeof unit i at time t if the unit is treated after τ

Synthetic Controls: Set-up

We want to **estimate the effect of the intervention on the treated units** for all time periods after τ: $(\alpha_{1,\tau+1}, \ldots, \alpha_{1T})$

$$\alpha_{1t} = Y^1_{1t} - Y^0_{1t} = Y_{1t} - Y^0_{1t},$$

Y_{1t} is the **observed outcome of the treated unit**

The challenge is to **find the counterfactual** Y^0_{1t}

Synthetic Controls: Implementation

We construct the **counterfactual as the weighted average** of the **outcomes of the donor pool**

$$Y_{1t}^{0} = \sum_{j} w_j^* Y_{jt}$$

- $w_j^* \in [0, 1]$ is the weight of donor unit
- $w_j^* \geq = 0 \ \forall j, \sum_j w_j^* \ \forall j$
- Y_{jt} is the outcome of donor unit j in time t

The optimal weights are the result of an **optimization procedure**

Synthetic Controls: Implementation

How do we **find the optimal weight vector** $W^* = (w_2^*, ..., w_{J+1}^*)'$?

- We have a **set of weights,** W, such that some (or zero) weight is placed on each potential donor unit.
- A different weight vector (W) implies a different synthetic control.
- Let X_1 be a $(k \times 1)$ **vector of pre-intervention characteristics** for the **treated unit**. Similarly, let X_0 be a $(k \times J)$ matrix which contains the **same variables for the unaffected units**.
- The goal is to find the **weight vector**, W^*, that brings the **weighed value of X_0 as close as possible to X_1**.

Synthetic Controls: Estimation

X can **include pre-treatment characteristics** as well as **pre-treatment outcomes**

We need to find two sets of weights:

- The weight vector W^* $\Rightarrow$ weight of each unit in the synthetic control
- V: diagonal weight matrix of each variable in predicting the synthetic control

Synthetic Controls: Estimation

Minimization problem

$$\|X_1 - X_0 W\| = \sqrt{(X_1 - X_0 W)' \, V \, (X_1 - X_0 W)},$$

- Letting v_m be the diagonal element relating to the *mth* covariate, then the weights $w_2^*, ..., w_{J+1}^*$ minimise:

$$\sum_{m=1}^{k} v_m \left(X_{1m} - \sum_{j=2}^{J+1} w_j X_{jm} \right)^2$$

- **Choice of *v*s** can be subjective or could be based on a **pre-treatment regression of *Y* on *X*** or some other algorithm.

Synthetic Controls: Estimation

This **procedure sounds daunting...**

but the **optimization** is usually **done by statistical software**

Jens Hainmueller has developed the synth package for Stata, Matlab and R

He also has a nice video showing how to implement this

Application 1: Abadie & Gardeazabal (2003)

Abadie & Gardeazabal (2003) provide the first well-known application of SC

They want to estimate the **effect of terrorism in the Basque country on growth**

Challenge: no other Spanish region followed the same trend

⇒ use **weighted average across Spanish regions** as **synthetic control group**

Basque Country vs. the Rest of Spain

TABLE 3—PRE-TERRORISM CHARACTERISTICS, 1960'S

	Basque Country (1)	Spain (2)
Real per capita GDP[a]	5,285.46	3,633.25
Investment ratio (percentage)[b]	24.65	21.79
Population density[c]	246.89	66.34
Sectoral shares (percentage)[d]		
Agriculture, forestry, and fishing	6.84	16.34
Energy and water	4.11	4.32
Industry	45.08	26.60
Construction and engineering	6.15	7.25
Marketable services	33.75	38.53
Nonmarketable services	4.07	6.97
Human capital (percentage)[e]		
Illiterates	3.32	11.66
Primary or without studies	85.97	80.15
High school	7.46	5.49
More than high school	3.26	2.70

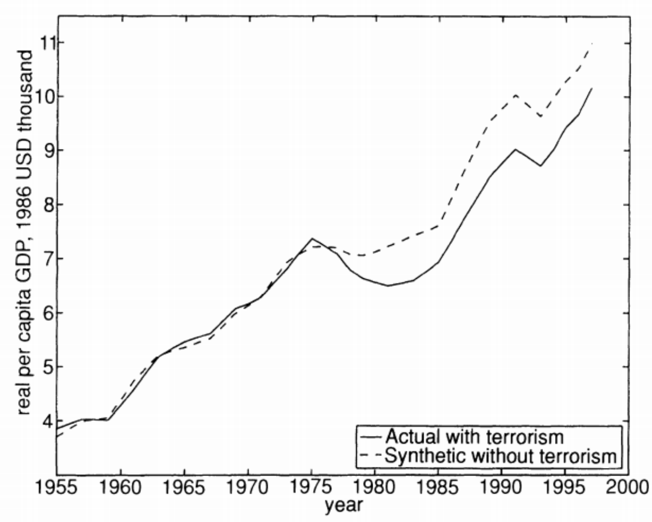
Basque Country vs. Synthetic control

After choice of optimal weights W^*, V^*: Catalonia:0.8508, Madrid: 0.1492

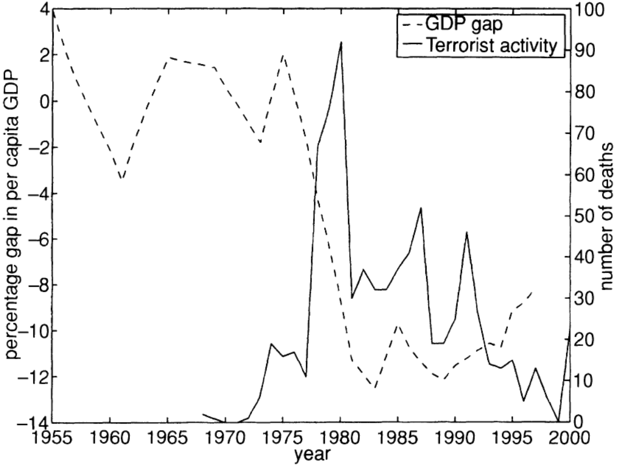
	Basque Country (1)	Spain (2)	"Synthetic" Basque Country (3)
Real per capita GDP[a]	5,285.46	3,633.25	5,270.80
Investment ratio (percentage)[b]	24.65	21.79	21.58
Population density[c]	246.89	66.34	196.28
Sectoral shares (percentage)[d]			
Agriculture, forestry, and fishing	6.84	16.34	6.18
Energy and water	4.11	4.32	2.76
Industry	45.08	26.60	37.64
Construction and engineering	6.15	7.25	6.96
Marketable services	33.75	38.53	41.10
Nonmarketable services	4.07	6.97	5.37
Human capital (percentage)[e]			
Illiterates	3.32	11.66	7.65
Primary or without studies	85.97	80.15	82.33
High school	7.46	5.49	6.92
More than high school	3.26	2.70	3.10

Basque Country vs. Synthetic control

Now use W^* to compute $Y^0_{1t} = \sum_j w^*_j Y_{jt}$



Estimated GDP vs. Terrorism



What about Unobservable Factors?

As with any **Diff-in-Diff**, **causal identification relies on the common trends assumption**

The outcomes could have diverged after τ for reasons other than terrorism

But this is less of an issue when

- we have a **long pre-treatment period**
- and match based on **pre-treatment outcomes**

$\Rightarrow$ not plausible that **factors that produce a tight fit** before would **diverge afterwards**

Inference

Conventional statistical inference is difficult because we typically **have two time series**

- 2T observations
- strong serial correlation and too few clusters

Alternative: permutation tests

- run **placebo SC** on all units in the donor pool
- compute the **treatment effect for each placebo**
- compare placebos to the estimated treatment effect
- compute **empirical p-value**

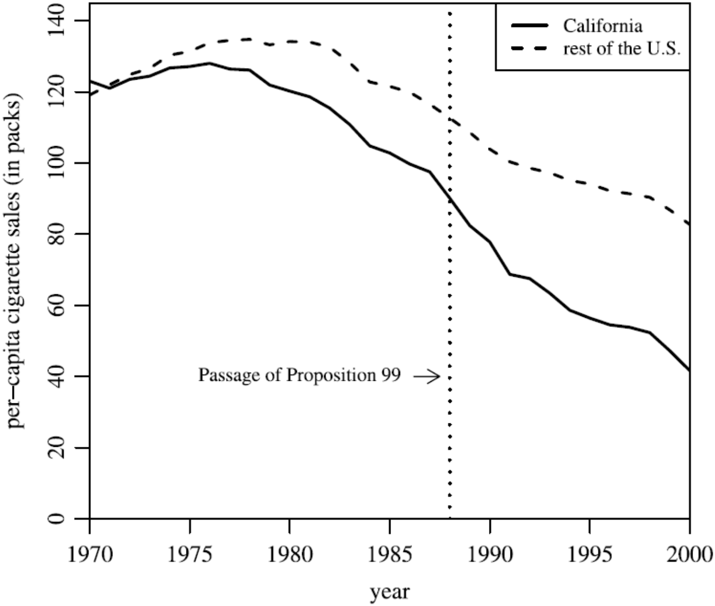
Second Example: Abadie *et al.* (2010)

Abadie *et al.* (2010) evaluate a **tobacco control program in California 1988**

Proposition 99

- increase in cigarette taxes by 25cent per pack
- information campaigns
- clean indoor-air campaigns

Cigarette Sales



Predictors for Choosing Weights

Table 1. Cigarette sales predictor means

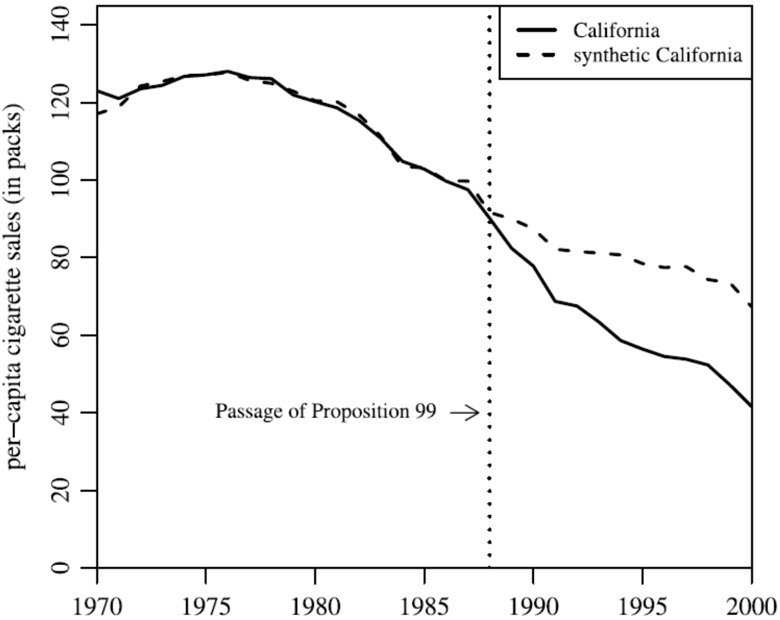
Variables	California Real	California Synthetic	Average of 38 control states
Ln(GDP per capita)	10.08	9.86	9.86
Percent aged 15–24	17.40	17.40	17.29
Retail price	89.42	89.41	87.27
Beer consumption per capita	24.28	24.20	23.75
Cigarette sales per capita 1988	90.10	91.62	114.20
Cigarette sales per capita 1980	120.20	120.43	136.58
Cigarette sales per capita 1975	127.10	126.99	132.81

Optimal Weights

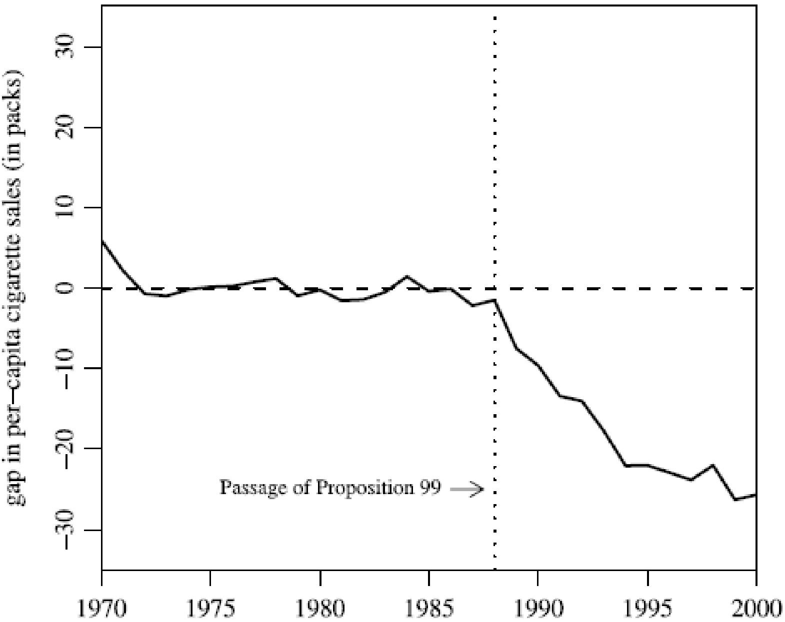
Table 2. State weights in the synthetic California

State	Weight	State	Weight
Alabama	0	Montana	0.199
Alaska	–	Nebraska	0
Arizona	–	Nevada	0.234
Arkansas	0	New Hampshire	0
Colorado	0.164	New Jersey	–
Connecticut	0.069	New Mexico	0
Delaware	0	New York	–
District of Columbia	–	North Carolina	0
Florida	–	North Dakota	0
Georgia	0	Ohio	0
Hawaii	–	Oklahoma	0
Idaho	0	Oregon	–
Illinois	0	Pennsylvania	0
Indiana	0	Rhode Island	0
Iowa	0	South Carolina	0
Kansas	0	South Dakota	0
Kentucky	0	Tennessee	0
Louisiana	0	Texas	0
Maine	0	Utah	0.334
Maryland	–	Vermont	0
Massachusetts	–	Virginia	0
Michigan	–	Washington	–
Minnesota	0	West Virginia	0
Mississippi	0	Wisconsin	0
Missouri	0	Wyoming	0

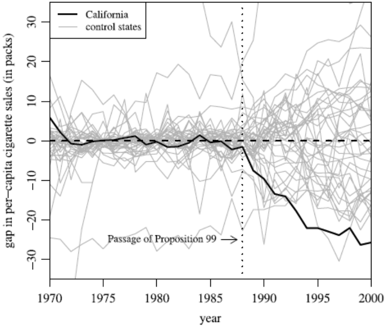
California vs. Synthetic Control

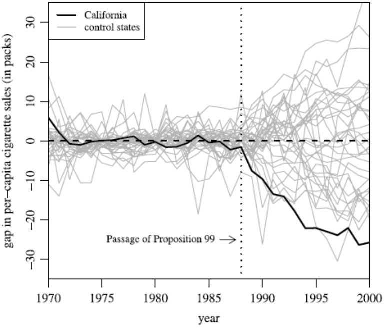


California vs. Synthetic Control



Permutation Test





Permutation Test

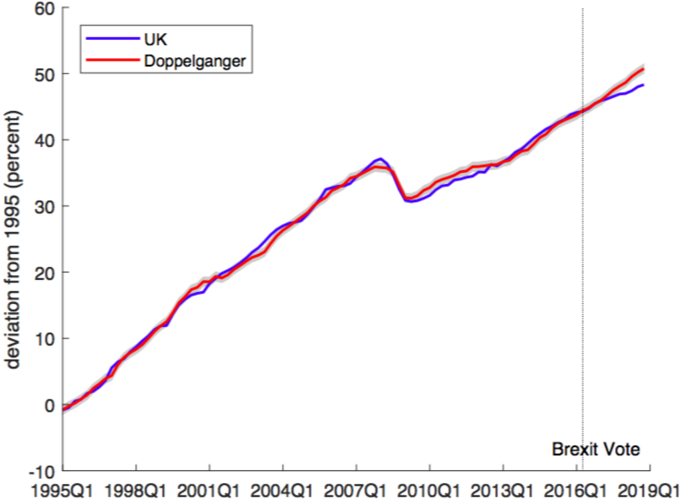
The **permutation test** reveals that **California is a clear outlier**

Based on the placebo treatment effects, it is possible to compute an **empirical p-value**

$$p = \frac{1+b}{1+N}$$

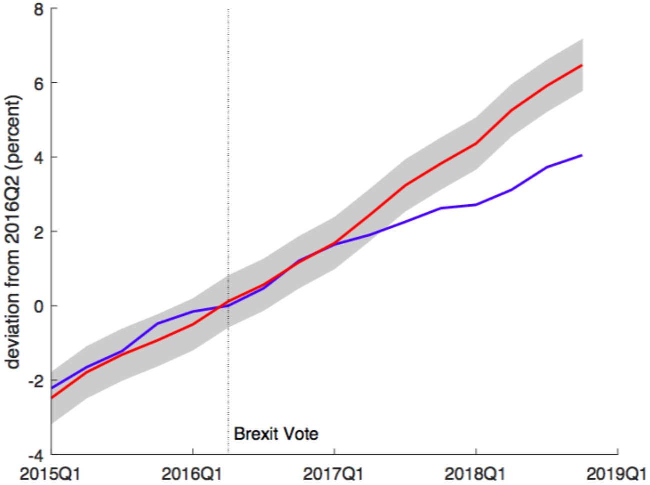
- $b \Rightarrow$ Number of placebo estimates larger in absolute value than our estimate
- $N \Rightarrow$ Number of placebo estimates

I Couldn't Resist Including This One



From: Born *et al.* (2019)

I Couldn't Resist Including This One



From: Born *et al.* (2019)

What's Interesting about this Study

Born *et al.* (2019) are very careful about **robustness checks**

- Conventional randomization inference
- Placebo Brexit vote dates
- Placebos with restricted donor pool

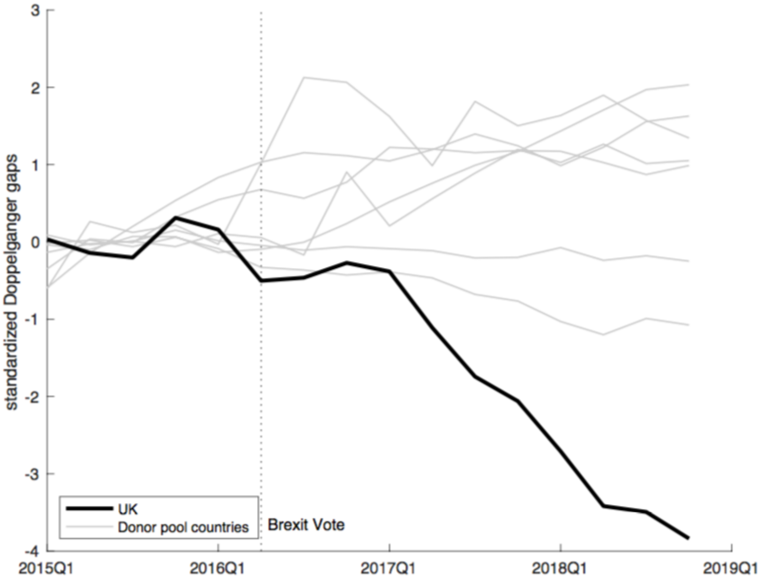
In addition: they look at **channels** and estimate an **expectation-augmented VAR**

Country Weights in Born *et al.* (2019)

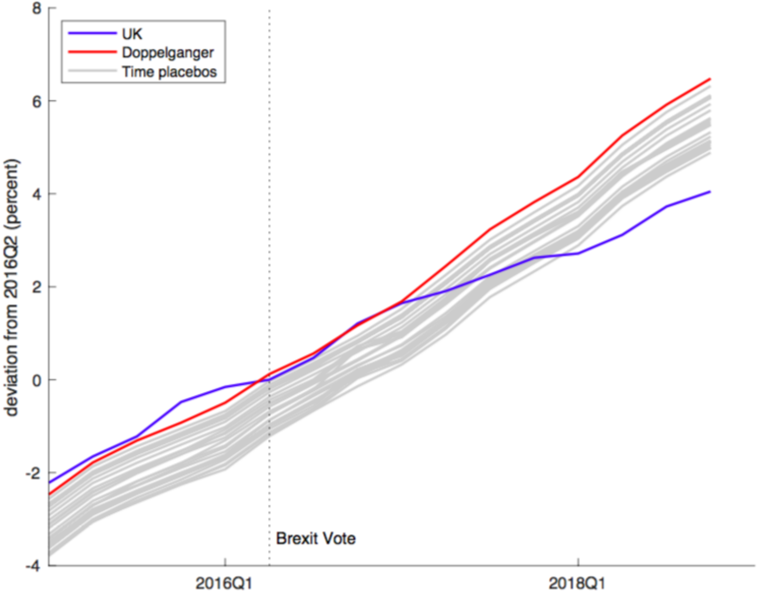
Table 2: Composition of the doppelganger: country weights

Australia	<0.01	Austria	<0.01	Belgium	<0.01	Canada	<0.01
Finland	<0.01	France	<0.01	Germany	0.05	Hungary	0.11
Iceland	0.01	Ireland	0.01	Italy	0.17	Japan	<0.01
Korea	<0.01	Luxembourg	<0.01	Netherlands	<0.01	New Zealand	0.14
Norway	<0.01	Portugal	<0.01	Slovak Republic	<0.01	Spain	<0.01
Sweden	<0.01	Switzerland	<0.01	United States	0.51		

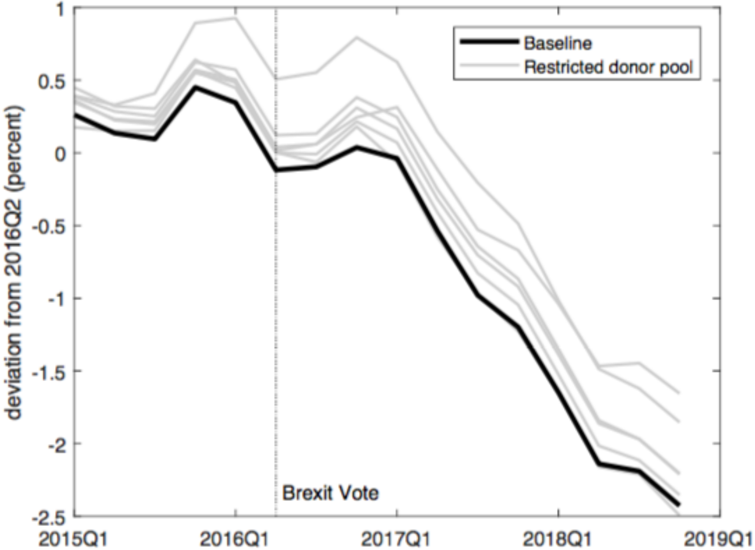
Conventional Randomization Checks



Placebos: Brexit Vote at Different Dates



Placebos: Leave out Important Donor Countries



Synthetic Controls: the Cookbook I

Follow the Standard Protocol

- Think and explain why there should be a **causal effect**
- Select a **donor pool** and **construct the counterfactual**
- **Report pre-treatment characteristics** for treatment and counterfactual
- Show the **main results graphically**
- Perform **permutation tests** and show them graphically

Synthetic Controls: the Cookbook II

More robustness checks

- Report counterfactuals **different matching periods**
- Perform placebo tests with **restricted donor pools**

Complement SC with another method

- Conventional DiD
- Time series models, etc etc

SC — Additional Readings

Abadie (forthcoming) has an excellent **overview article** in the JEL

Refinements

- Abadie & L'Hour (2019) develop a machine learning procedure for datasets with many units in the donor pool (and the problem of **multiple optimal synthetic controls**)
- Ferman & Pinto (2016) shows under what conditions causal inference is valid even if the **pre-treatment match is not perfect.** Ben-Michael *et al.* (2018) develop an augmented estimator that deals with this problem.
- Botosaru & Ferman (2019) derive bounds on SC estimates if **covariates are not balanced**
- Kaul *et al.* (2015) show why researchers should **not include all lagged outcomes** in the matching algorithm

New Developments in SC

Analyzing multiple case studies

- essentially a mix between SC and event studies
- examples: Acemoglu *et al.* (2016), Xu (2017)
- Methods paper on synthetic difference-in-differences: Arkhangelsky *et al.* (2019)

Synthetic Control Meets Machine Learning

- Machine learning algorithms (and larger datasets) can improve the choice of predictors
- Most new techniques are based on matrix completion methods (Athey *et al.*, 2018)

References I

Abadie, Alberto. forthcoming. Using Synthetic Controls: Feasibility, Data Requirements, and Methodological Aspects. *Journal of Economic Literature*.

Abadie, Alberto, & Gardeazabal, Javier. 2003. The Economic Costs of Conflict: A Case Study of the Basque Region. *American Economic Review*, **93**(1), 113–132.

Abadie, Alberto, & L'Hour, Jérémy. 2019. A Penalized Synthetic Control Estimator for Disaggregated Data. *Harvard University, mimeo*.

Abadie, Alberto, Diamond, Alexis, & Hainmueller, Jens. 2010. Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California's Tobacco Control Program. *Journal of the American Statistical Association*, **105**(490), 493–505.

Acemoglu, Daron, Johnson, Simon, Kermani, Amir, Kwak, James, & Mitton, Todd. 2016. The value of connections in turbulent times: Evidence from the United States. *Journal of Financial Economics*, **121**(2), 368–391.

Arkhangelsky, Dmitry, Athey, Susan, Hirshberg, David A, Imbens, Guido W, & Wager, Stefan. 2019. Synthetic Difference In Differences. February.

Athey, Susan, Bayati, Mohsen, Doudchenko, Nikolay, Imbens, Guido, & Khosravi, Khashayar. 2018. Matrix Completion Methods for Causal Panel Data Models. *NBER Working Papers*, Oct.

Ben-Michael, Eli, Feller, Avi, & Rothstein, Jesse. 2018. The Augmented Synthetic Control Method.

Born, Benjamin, Müller, Gernot J., Schularick, Moritz, & Sedlacek, Petr. 2019. The Cost of Economic Nationalism: Evidence from the Brexit Experiment. *Economic Journal*.

Botosaru, Irene, & Ferman, Bruno. 2019. On the role of covariates in the synthetic control method. *The Econometrics Journal*, **22**(2), 117–130.

Ferman, Bruno, & Pinto, Cristine. 2016. Synthetic Controls with Imperfect Pre-Treatment Fit.

Kaul, Ashok, Klößner, Stefan, Pfeifer, Gregor, & Schieler, Manuel. 2015. Synthetic Control Methods: Never Use All Pre-Intervention Outcomes Together With Covariates. Mar.

Xu, Yiqing. 2017. Generalized Synthetic Control Method: Causal Inference with Interactive Fixed Effects Models. *Political Analysis*, **25**(01), 57–76.